Cuaderno de notas de trabajo

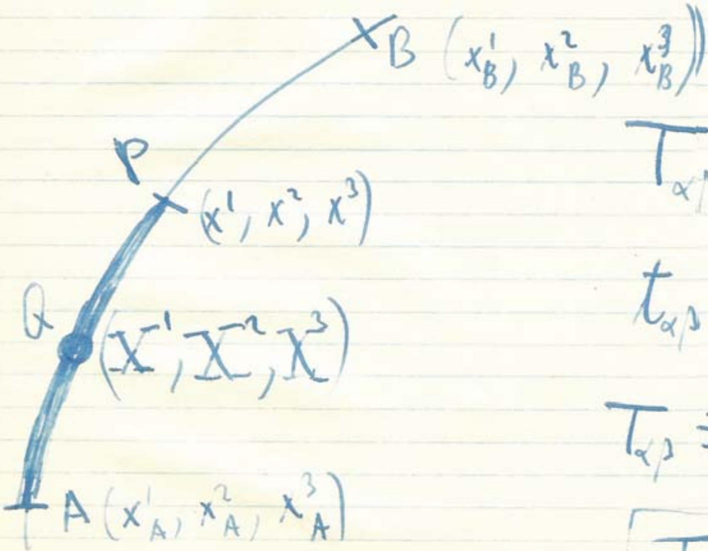
Carlos Graef Fernández

Cuaderno Sn 4

Consideraciones en Tres Dimensiones.

Principio Variacional.

$\alpha = 1, 2, 3$
$\beta = 1, 2, 3$



$$T_{\alpha\beta}(X^1, X^2, X^3)$$

$$t_{\alpha\beta} = (x^1, x^2, x^3)$$

$$T_{\alpha\beta} \equiv t_{\alpha\beta}$$

$$\boxed{T_{\alpha\beta} = T_{\beta\alpha}}$$

Acción A.

$$\boxed{A = \int_A^B \left\{ \int_A^P T_{\alpha\beta} \, d\mathcal{X}^{\beta} \right\} dx^{\alpha}}$$

$$\boxed{\delta A = 0}$$

$$B_\alpha = \int_A^P T_{\alpha\beta}\, dX^\beta$$

$$\delta B_\alpha = \int_A^P \left[\frac{\partial T_{\alpha\beta}}{\partial X^\gamma} \delta X^\gamma dX^\beta + T_{\alpha\beta}\, d(\delta X^\beta) \right]$$

$$\delta B_\alpha = T_{\alpha\beta}\, \delta X^\beta \Big|_A^P + \int_A^P \left[\frac{\partial T_{\alpha\beta}}{\partial X^\gamma} \delta X^\gamma dX^\beta - \left\{ \frac{\partial T_{\alpha\beta}}{\partial x^\delta} dX^\delta + \frac{\partial T_{\alpha\beta}}{\partial t} dt \right\} \delta X^\beta \right]$$

$$\boxed{\delta B_\varrho = t_{\varrho\beta}\, \delta x^\beta + \int_A^P \left[\frac{\partial T_{\varrho\sigma}}{\partial X^\tau} - \frac{\partial T_{\varrho\tau}}{\partial X^\sigma} \right] \delta X^\tau dX^\sigma - \int_A^P \frac{\partial T_{\varrho\beta}}{\partial t} \delta X^\beta dt}$$

$$\cancel{dB_\alpha = T_{\alpha\beta}}$$

$$\boxed{\cancel{dB_\alpha = t_{\alpha\beta}\, dx^\beta}}$$

$$\boxed{dB_\alpha = T_{\alpha\beta}\, dX^\beta}$$

$$A = \int_A^B B_\alpha \, dx^\alpha$$

$$\delta A = \int_A^B \delta B_\alpha \, dx^\alpha + \int_A^B B_\alpha \, d(\delta x^\alpha)$$

$$\delta A = \int_A^B \left[t_{\alpha\beta}\, \delta x^\beta dx^\alpha + \left\{ \int_A^P \left(\frac{\partial T_{\alpha\sigma}}{\partial X^\tau} - \frac{\partial T_{\alpha\tau}}{\partial X^\sigma} \right) \delta X^\tau dX^\sigma \right\} dx^\alpha - \int_A^P \frac{\partial T_{\alpha\beta}}{\partial t} \delta X^\beta dt \, dx^\alpha \right]$$

$$B$$

$$\delta A = \int_A^B \left[\cancel{t_{\alpha\beta}\,\delta x^\beta} + \left\langle \int_A^P \left\{ \frac{\partial T_{\alpha\sigma}}{\partial X^\tau} - \frac{\partial T_{\alpha\tau}}{\partial X^\sigma} \right\} \delta X^\tau dX^\sigma \right\rangle - \int_A^P \frac{\partial T_{\alpha\beta}}{\partial t} \delta X^\beta dt \right] dx^\alpha$$

$$- \cancel{t_{\alpha\beta}\, dx^\beta \delta x^\alpha}$$

por la simetría de $t_{\alpha\beta}$

$$\cancel{\delta A = \int_A^B \left[\frac{\partial T_{\alpha\sigma}}{\partial X^\tau} - \frac{\partial T_{\alpha\tau}}{\partial X} \right.}$$

$$\boxed{\delta A = \int_A^B \left[\int_A^P \left\langle \left\{ \frac{\partial T_{\alpha\sigma}}{\partial X^\tau} - \frac{\partial T_{\alpha\tau}}{\partial X^\sigma} \right\} \dot{X}^\sigma - \frac{\partial T_{\alpha\tau}}{\partial t} \right\rangle \delta X^\tau dt \right] dx^\alpha}$$

$$\ddot{x} = -\frac{Mx}{r^3} - \frac{Mxv^2}{r^3} + \frac{2M\dot{x}\dot{r}}{r^2}$$

$$\ddot{y} = -\frac{My}{r^3} - \frac{Myv^2}{r^3} + \frac{2M\dot{y}\dot{r}}{r^2}$$

$$\ddot{z} = -\frac{Mz}{r^3} - \frac{Mzv^2}{r^3} + \frac{2M\dot{z}\dot{r}}{r^2}$$

$$e^{ijkl} a_k v_l = \tau^{ij} \qquad a_k = (t'', -x'', -y'', -z'')$$

$$v_l = (t', -x', -y', -z')$$

$$a_k = (t'', -x'', -y'', -z'')$$
$$v_l = (t', -x', -y', -z')$$

$$a_k = (t'', -x'', -y'', -z'')$$

$$v_l = (t', -x', -y', -z')$$

$\boxed{e^{ijkl} a_k v_l}$

0	$y''z' - y'z''$	$x'z'' - x''z'$	$x''y' - x'y''$
$y'z'' - y''z'$	0	$-t''z' + t'z''$	$-t'y'' + t''y'$
$x''z' - x'z''$	$+t''z' - t'z''$	0	$-t''x' + t'x''$
$x'y'' - x''y'$	$-t''y' + t'y''$	$+t''x' - t'x''$	0

$$a_k = - B_{ij,k} v^i v^j$$

$$a_k v_l = - B_{ij,k} v^i v^j v_l$$

$$a_k v_l = - B_{pq,k} v^p v^q v_l$$

$$e^{ijkl} a_k v_l = - e^{ijkl} B_{pq,k} v_l v^p v^q$$

$$\boxed{\text{Calcular: } e^{ijkl} B_{pq,k} v_l}$$

$\boxed{e^{ijkl} B_{pq,k} V_l}$

0	$+B_{pq,3}V_4 - B_{pq,4}V_3$	$-B_{pq,2}V_4 + B_{pq,4}V_2$	$+B_{pq,2}V_4 - B_{pq,4}V_2$
$-B_{pq,3}V_4 + B_{pq,4}V_3$	0	$+B_{pq,1}V_4 - B_{pq,4}V_1$	$-B_{pq,1}V_3 + B_{pq,3}V_1$
$+B_{pq,2}V_4 - B_{pq,4}V_2$	$-B_{pq,1}V_4 + B_{pq,4}V_1$	0	$+B_{pq,1}V_2 - B_{pq,2}V_1$
$-B_{pq,2}V_3 + B_{pq,3}V_2$	$+B_{pq,1}V_3 - B_{pq,3}V_1$	$-B_{pq,1}V_2 + B_{pq,2}V_1$	0

Acción B.

$$B = \int_A^B \left\{ \int_A^P \Delta_{\alpha\beta} \, d\bar{X}^\beta \right\} dx^\alpha$$

$\underline{\underline{\delta B}}$

$$C_\alpha = \int_A^P \Delta_{\alpha\beta}\, d\overline{X}^\beta$$

$$\delta C_\alpha = \cancel{\Delta_{\alpha\beta}\, \delta \overline{X}^\beta} \quad \Delta_{\alpha\beta}\, \delta x^\beta$$

$$B = \int_A^B C_\alpha\, dx^\alpha$$

$$\delta B = \int_A^B \left[\Delta_{\alpha\beta}\, \delta x^\beta dx^\alpha + C_\alpha\, d(\delta x^\alpha)\right]$$

$$\delta B = \int_A^B \left[\Delta_{\alpha\beta}\, \delta x^\beta dx^\alpha - dC_\alpha\, \delta x^\alpha\right]$$

$$\delta B = \int_A^B \left[\Delta_{\alpha\beta}\, \delta x^\beta dx^\alpha - \Delta_{\alpha\beta}\, dx^\beta \delta x^\alpha\right] = 0$$

$$\delta \int_A^B \cancel{\int_A^P \sqrt{\Delta_{pq}\, d\overline{X}^p d\overline{X}^q} = \int_A^B \left[\Delta_{pq} V^q \delta x^p - \int_A^P \Delta_{pq} A^q \delta \overline{X}^p ds\right.}$$

$$\int_A^B \left\langle \int_A^P \sqrt{\Delta_{pq}\, d\overline{X}^p d\overline{X}^q} \right\rangle ds$$

$$I \equiv \int_0^1 t^{\beta-1}(1-t)^{\gamma-\beta-1}(1-xt)^{-\alpha}\,dt$$

$$I \equiv \int_0^1 t^{\beta-1}(1-t)^{\gamma-\beta-1}\Big[1+\frac{\alpha}{1!}xt+\frac{\alpha(\alpha+1)}{1\cdot 2}x^2t^2+\frac{\alpha(\alpha+1)(\alpha+2)}{1\cdot 2\cdot 3}x^3t^3+\frac{\alpha(\alpha+1)(\alpha+2)(\alpha+3)}{1\cdot 2\cdot 3\cdot 4}x^4t^4+\frac{\alpha(\alpha+1)(\alpha+2)(\alpha+3)(\alpha+4)}{1\cdot 2\cdot 3\cdot 4\cdot 5}x^5t^5+\ldots\Big]dt$$

$$+\frac{\alpha(\alpha+1)(\alpha+2)\cdots(\alpha+r-1)}{1\cdot 2\cdot 3\cdots r}x^r t^r+\cdots$$

$$I \equiv \sum_{r=0}^{\infty}\int_0^1 t^{\beta+r-1}(1-t)^{\gamma-\beta-1}\frac{\alpha(\alpha+1)\cdots(\alpha+r-1)}{1\cdot 2\cdots r}x^r\,dt$$

$$\cancel{I = \int_0^1 t^{\beta-1}(1-t)^{\gamma-\beta-1}\Big[1+}$$

$$I \equiv \sum_{r=0}^{\infty} x^r \frac{\alpha(\alpha+1)(\alpha+2)\cdots(\alpha+r-1)}{1\cdot 2\cdot 3\cdots r} \int_0^1 t^{\beta+r-1}(1-t)^{\gamma-\beta-1}\,dt$$

& $$\int_0^1 t^{\beta+r-1}(1-t)^{\gamma-\beta-1}\,dt$$

$$\cancel{t = \sin\theta}$$

$$\cancel{dt = \cos\theta\,d\theta}$$

$$\cancel{\int_0^1 \sin^{\beta+r-1}\theta \cos\theta^{\gamma-\beta}\,d\theta = \Gamma}$$

$$t = \sin^2\theta$$

$$dt = 2\sin\theta\cos\theta\,d\theta$$

$$2\int_0^{\frac{\pi}{2}} \sin\theta^{2\beta+2r-1}\cos^{2\gamma-2\beta-1}\,d\theta$$

$$= \frac{\Gamma(\beta+r)\,\Gamma(\gamma-\beta)}{\Gamma(\gamma+r)} = B(\beta+r,\gamma-\beta)$$

$$I = \sum_{0} x^r \frac{\alpha(\alpha+1)(\alpha+2)\cdots(\alpha+r-1)}{1\cdot 2\cdot 3\cdots r} B(\beta+r,\gamma-\beta)$$

$$I = \sum_0^\infty x^r \frac{\alpha(\alpha+1)(\alpha+2)\cdots(\alpha+r-1)}{1\cdot 2\cdot 3\cdots r} \frac{\Gamma(\beta+r)\,\Gamma(\gamma-\beta)}{\Gamma(\gamma+r)}$$

$$I = \sum_0^\infty x^r \frac{\alpha(\alpha+1)(\alpha+2)\cdots(\alpha+r-1)}{1\cdot 2\cdot 3\cdots r} \frac{(\beta+r-1)(\beta+r-2)\cdots\beta}{(\gamma+r-1)(\gamma+r-2)\cdots\gamma} \frac{\Gamma(\beta)\Gamma(\gamma-\beta)}{\Gamma(\gamma)}$$

$$I = \sum_r x^r$$

$$I = \frac{\Gamma(\beta)\,\Gamma(\gamma-\beta)}{\Gamma(\gamma)} \sum_r \frac{\alpha(\alpha+1)\cdots(\alpha+r-1)(\beta)(\beta+1)\cdots(\beta+r-1)}{1\cdot 2\cdots r\;\gamma(\gamma+1)\cdots(\gamma+r-1)} x^r$$

$$\boxed{I = B(\beta,\gamma-\beta)\, F(\alpha,\beta,\gamma,x)}$$

$$\int_0^1 (t$$

$$\frac{x}{x-1} = \xi$$

$$\frac{x}{1} = \frac{\xi}{\xi-1}$$

$$\int_0^1 t^{\beta-1}(1-t)^{\gamma-\beta-1}(1-xt)^{-\alpha}\,dt = B(\beta,\gamma-\beta)\,F(\alpha,\beta,\gamma,x)$$

$$t \to (1-t)$$
$$1-t \to t$$

$$(-1)\int_1^0 (1-t)^{\beta-1} t^{\gamma-\beta-1}(1-x+tx)^{-\alpha}\,dt =$$

$$+\int_0^1 t^{\gamma-\beta-1}(1-t)^{\beta-1}(1-x)^{-\alpha}\left(1+\frac{xt}{1-x}\right)^{-\alpha} dt$$

$$= (1-x)^{-\alpha}\, B(\gamma-\beta,\beta)\, F\left(\alpha,\gamma-\beta,\gamma,-\frac{x}{1-x}\right)$$

La integral $\int_0^1 t^{\beta-1}(1-t)^{\gamma-\beta-1}(1-xt)^{-\alpha}\,dt$

es invariante ante cambios de la variable de integración. $t=\tau-1$

La función hipergeométrica tiene la siguiente invariancia

$$F(\alpha,\beta,\gamma,x)=F(\beta,\alpha,\gamma,x)$$

$$\int_0^1 t^{\beta-1}(1-t)^{\gamma-\beta-1}(1-xt)^{-\alpha}\,dt = B(\beta,\gamma-\beta)F(\alpha,\beta,\gamma,x)$$

$$-\int_1^0 (1-\tau)^{\beta-1}\tau^{\gamma-\beta-1}(1-x+\tau x)^{-\alpha}\,d\tau = B(\beta,\gamma-\beta)F(\alpha,\beta,\gamma,x)$$

$$\int_0^1 \tau^{\gamma-\beta-1}(1-\tau)^{\beta-1}(1-x+\tau x)^{-\alpha}\,d\tau =$$

$$(1-x)^{-\alpha}\int_0^1 \tau^{\gamma-\beta-1}(1-\tau)^{\beta-1}\left(1+\frac{x}{1-x}\tau\right)^{-\alpha}d\tau$$

$$\int_0^1 t^{\beta-1}(1-t)^{\gamma-\beta-1}(1-xt)^{-\alpha}\,dt = B(\beta,\gamma-\beta)F(\alpha,\beta,\gamma,x)$$

$$\int_0^1 t^{\alpha-1}(1-t)^{\gamma-\alpha-1}(1-xt)^{-\beta}\,dt = B(\alpha,\gamma-\alpha)F(\beta,\alpha,\gamma,x)$$

$$F(\alpha,\beta,\gamma,x) = \frac{B(\alpha,\gamma-\alpha)}{B(\beta,\gamma-\beta)}F(\beta,\alpha,\gamma,x)$$

$$\cancel{B(\beta,\gamma-\beta)\, F(\alpha,\beta,\gamma,x) = (1-x)^{-\alpha} F(\gamma,\beta}$$

$$\cancel{B(\beta,\gamma-\beta)}\, F(\alpha,\beta,\gamma,x) = (1-x)^{-\alpha}\, \cancel{B(\gamma-\beta,\beta)}\, F\left(\alpha,\gamma-\beta,\cancel{\beta},\frac{x}{x-1}\right)$$

$$B(\beta,\gamma-\beta) = B(\gamma-\beta,\beta)$$

$$F(\alpha,\beta,\gamma,x) = (1-x)^{-\alpha} F\left(\alpha,\gamma-\beta,\cancel{\beta},\frac{x}{x-1}\right)$$

$$F(\alpha,\beta,\gamma,x) = (1-x)^{-\beta} F\left(\beta,\gamma-\alpha,\gamma,\frac{x}{x-1}\right)$$

$$(1-x)^{-\alpha} F\left(\alpha,\gamma-\beta,\gamma,\frac{x}{x-1}\right) = (1-x)^{-\beta} F\left(\beta,\gamma-\alpha,\gamma,\frac{x}{x-1}\right)$$

$$\frac{x}{x-1} = \frac{\xi}{1} \qquad x-1 = \frac{1}{\xi-1}$$

$$\left(\frac{1}{1-\xi}\right)^{-\alpha} F(\alpha,\gamma-\beta,\gamma,\xi) = \left(\frac{1}{1-\xi}\right)^{-\beta} F(\beta,\gamma-\alpha,\gamma,\xi)$$

$$F(\alpha,\gamma-\beta,\gamma,\xi) = (1-\xi)^{\beta-\alpha} F(\beta,\gamma-\alpha,\gamma,\xi)$$

$\gamma-\beta = c$ $\quad \cancel{\beta = c-b}$

$\gamma = b$

$\gamma-\beta = b \qquad \beta = c-b$

$\gamma = c$

$$F(\alpha, b, c, \xi) = (1-\xi)^{c-b-\alpha} F(c-b, c-\alpha, c, \xi)$$

$$F(\alpha, \beta, \gamma, x) = (1-x)^{\gamma-\beta-\alpha} F(\cancel{\gamma-\beta, \gamma-\alpha}, \gamma, x)$$

PRINCIPIO VARIACIONAL.

Definiciones:

1) $$\boxed{W_i = \int_{A\,\Gamma}^{B} h_{ij}\, dx^j}$$

Camino de integración

W_i es un vector constante que depende del camino de integración.

$$W_i = W_i(\Gamma).$$

2) $$\boxed{V = \int_A^B \left\{ \int_A^B h_{ij}\, dx^j \right\} dx^i = \int_A^B W_i\, dx^i}$$

Energía Potencial

$$\boxed{\delta W_i \not\equiv 0.}$$

$$\delta W_i = \int_A^B \left[\frac{\partial h_{ij}}{\partial x^k}\delta x^k dx^j + h_{ij}\, d(\delta x^j)\right]$$

$$\delta W_i = h_{ij}\,\delta x^j \Big|_A^B + \int_A^B \left[\frac{\partial h_{ij}}{\partial x^k}\delta x^k dx^j - \frac{\partial h_{ij}}{\partial x^p} dx^p\,\delta x^j\right]$$

$$\boxed{\delta W_i = \int_A^B \left[\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{ik}}{\partial x^j}\right]\delta x^k dx^j}$$

$$\boxed{V = \int_A^B W_i \, dx^i}$$

$$\delta V = \int_A^B \left[\delta W_i \, dx^i + W_i \, d(\delta x^i) \right]$$

$$\delta V = W_i \, \delta x^i \Big|_A^B + \int_A^B \left[\delta W_i \, dx^i - \overset{\text{nula}}{(dW_i)} \, \delta x^i \right]$$

$$\delta V = \int_A^B \left[\left\{ \int_A^B \left[\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{ik}}{\partial x^j} \right] \delta x^k \, dx^j \right\} \right] dx^i$$

$$\boxed{\delta V = \int_A^B \left\{ \int_A^B \left(\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{ik}}{\partial x^j} \right) \delta x^k \, dx^j \right\} dx^i}$$

V. Definiciones:

4) $$\boxed{T = \int_A^B \left\{ \int_A^B \sqrt{\Delta_{pq}\, dx^p\, dx^q} \right\} \sqrt{\Delta_{mn}\, dx^m\, dx^n}}$$

Energía cinética T

4) $$\boxed{T = \int_A^B s \sqrt{\Delta_{mn}\, dx^m\, dx^n}}$$

3) $$\boxed{s = \int_A^B \sqrt{\Delta_{pq}\, dx^p\, dx^q}}$$

$$\boxed{s = \int_A^B \sqrt{\Delta_{pq}\, dx^p dx^q}}$$

$$\delta s = \int_A^B \frac{\Delta_{pq}\, dx^p\, d(\delta x^q)}{ds} = \int_A^B \Delta_{pq}\, v^p\, d(\delta x^q)$$

$$\delta s = \Delta_{pq}\, v^p\, \delta x^q \Big|_A^B - \int_A^B \Delta_{pq}\, a^p\, \delta x^q\, ds$$

$$\boxed{\delta s = - \int_A^B \Delta_{pq}\, a^p\, \delta x^q\, ds}$$

$$T = \int_A^B s \sqrt{\Delta_{mn}\, dx^m\, dx^n}$$

$$\delta T = -\int_A^B \left[\left\{ \int_A^B \Delta_{pq}\, a^p\, \delta x^q\, ds \right\} \right] ds + \cancel{s\, \Delta_{mn}\, a^m}\; s\, \frac{\Delta_{mn}\, dx^m\, d(\delta x^n)}{ds}$$

$$\delta T = \int_A^B \left\langle \left[-\left\{ \int_A^B \Delta_{pq}\, a^p\, \delta x^q\, ds \right\} \right] ds + s\, \Delta_{mn}\, v^m\, d(\delta x^n) \right\rangle$$

$$\cancel{\delta T = \int_A^B \left\langle \left[-\left\{ \int_A^B \Delta_{pq}\, a^p\, \delta x^q\, ds \right\} \right] ds -}$$

$$P_n'(\mu) = \sum_{0}^{n-1} B_m P_m(\mu)$$

$$B_m(\mu) = \frac{2m+1}{2}\int_{-1}^{+1} P_m(\mu)\, P_n'(\mu)\, d\mu$$

$$B_m(\mu) = \frac{2m+1}{2}\left[P_m(\mu)\, P_n(\mu) \Big|_{-1}^{+1} - \int_{-1}^{+1} \right.$$

$$B_m(\mu) = \frac{2m+1}{2}\left[P_m(\mu)\, P_n'(\mu) \right.$$

$$P_n'(\mu) = \sum_0^{n-1} B_m P_m(\mu) \qquad m \leq n-1$$

$$B_m = \frac{2m+1}{2} \int_{-1}^{+1} P_m(\mu) P_n'(\mu) d\mu = \frac{2m+1}{2} \int_{-1}^{+}$$

$$B_n = \frac{2m+1}{2} \left[P_m(\mu) P_n(\mu) \Big|_{-1}^{+1} - \int_{-1}^{+1} P_m'(\mu) P_n(\mu) d\mu \right]$$

cero porque $m \leq n-1$

$$B_m = \frac{2m+1}{2} \left[1 - (-1)^{m+n} \right]$$

$m = n-1 \qquad \frac{2n-2+1}{2} = \frac{2n-1}{2} \qquad \underline{2n-6+1} \qquad 2n-9$

$m = n-3$

$$B_m = (2n-1) P_{n-1}(\mu) + (2n-5) P_{n-3} + (2n-9) P_{n-5} + \ldots$$

$$P_{n+1}'(\mu) = (2n+1) P_n(\mu) + (2n-3) P_{n-2} + (2n-7) P_{n-4} + \ldots$$

$$P_{n-1}'(\mu) = \qquad (2n-3) P_{n-2} + (2n-7) P_{n-4} + \ldots$$

$$P_{n+1}'(\mu) - P_{(n-1)}'(\mu) = (2n+1) P_n(\mu)$$

For $n = 0$

$$P_1'(\mu) = P_0(\mu)$$

$$\mu P'_{n+1}(\mu) = (n+1)\, P_{n+1}(\mu) + (2n-1)\, P_{n-1}(\mu) + (2n-5)\, P_{n-3}(\mu) + (2n-9)\, P_{n-5}(\mu)$$

$$\mu P'_{n-1}(\mu) = (n-1)\, P_{n-1}(\mu) + (2n-5)\, P_{n-3}(\mu) + (2n-9)\, P_{n-5}(\mu)$$

$$\mu \left[P'_{n+1}(\mu) - P'_{n-1}(\mu)\right] = (n+1)\, P_{n+1}(\mu) + n\, P_{n-1}(\mu)$$

$$P'_{n+1}(\mu) - P'_{n-1}(\mu) = (2n+1)\, P_n(\mu)$$

$$\cancel{\mu (2n+1) P_n(\mu) = (n+1)\, P_{n+1}(\mu)}$$

$$(n+1)\, P_{n+1}(\mu) - (2n+1)\mu\, P_n(\mu) + n\, P_{n-1}(\mu) = 0$$

$$P_1(\mu) - \mu P_0(\mu) = 0$$

$$x'^i = f\, x^i$$

$$\cancel{dx'^i = f\, dx^i + x^i \ldots}$$

$$dx'^i = f\, dx^i + x^i \frac{\partial f}{\partial x^j}\, dx^j$$

$$\Delta_{pq}\, dx'^p dx'^q = f\, \Delta_{rq}\, dx^p dx^q + \Delta_{pq} x^r x^q \frac{\partial f}{\partial x^j} dx^j \frac{\partial f}{\partial x^k} dx^k$$

$$dx'^i = f\, dx^i$$

$$\left(\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i}\right) \frac{dx^j}{ds} = T_{ik}$$

$$T_{ik} = \left(\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i}\right)\frac{dx^j}{ds}$$

$$S(T_{ik}) = \left\{ \frac{1}{2}\frac{\partial h_{ij}}{\partial x^k}\frac{dx^j}{ds} + \frac{1}{2}\frac{\partial h_{ik}}{\partial x^j}\frac{dx^j}{ds} - \frac{1}{2}\frac{\partial h_{jk}}{\partial x^i} \right.$$

$$\left. - \frac{1}{2}\frac{\partial h_{ji}}{\partial x^k}\frac{dx^j}{ds} \right.$$

$$T_{ik} = \left(\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i}\right)\frac{dx^j}{ds}$$ ¡antisimétrico!

$$S(T_{ik}) = \cancel{\left[\frac{1}{2}\frac{\partial h_{ij}}{\partial x^k}\frac{dx^j}{ds} + \frac{1}{2}\frac{\partial h_{kj}}{\partial x^i}\frac{dx^j}{ds} - \frac{1}{2}\frac{\partial h_{jk}}{\partial x^i}\frac{dx^j}{ds} - \frac{1}{2}\frac{\partial h_{ji}}{\partial x^k}\frac{dx^j}{ds}\right]}$$

$$T_{ik} = -T_{ki}$$

$$a_i v_k - a_k v_i \quad — \quad T_{ik}$$

$$(a_i v_k - a_k v_i)\, v^k = a_i \qquad T_{ik} v^k = a_i$$

$$a_i v_k - a_k v_i - T_{ik}$$

$$v^i v_i = 0 \qquad \hat{v} \perp \hat{v}$$

$$\frac{dv^i}{ds} v_i + v^i \frac{dv_i}{ds} = 0$$

$$v^i a_i = - \frac{dv^i}{ds} v_i$$

$$S_{ik} = \frac{1}{2}\left(\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i}\right)v^j + \frac{1}{2}\left(\frac{\partial h_{kj}}{\partial x^i} - \frac{\partial h_{ji}}{\partial x^k}\right)v^j$$

$$A_{ik} = \frac{1}{2}\left(\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i}\right)v^j - \frac{1}{2}\left(\frac{\partial h_{kj}}{\partial x^i} - \frac{\partial h_{ji}}{\partial x^k}\right)v^j$$

$$S_{ik} = \frac{1}{2}\left(\cancel{\frac{\partial h_{ij}}{\partial x^k}} - \cancel{\frac{\partial h_{jk}}{\partial x^i}} + \cancel{\frac{\partial h_{kj}}{\partial x^i}} - \cancel{\frac{\partial h_{ji}}{\partial x^k}}\right)v^j$$

$$A_{ik} = \frac{1}{2}\left(\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i} - \frac{\partial h_{kj}}{\partial x^i} + \frac{\partial h_{ji}}{\partial x^k}\right)v^j$$

$$A_{ik} = \left(\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i}\right)v^j$$

$$a_i v_k - a_k v_i - \left(\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i}\right) v^j = T_{ik}$$

$$T_{ik} = -T_{ki}$$

$$T_{ik} v^k = a_i - \left(\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i}\right) v^j v^k = 0$$

$$T_{ik} v^k = 0$$

$$T_{ik} v^i = 0$$

$$\left(\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i}\right) v^j v^k = a_i$$

$$\frac{\partial a_i}{\partial x^p} = \left(\frac{\partial^2 h_{ij}}{\partial x^k \partial x^p} - \frac{\partial^2 h_{jk}}{\partial x^i \partial x^p}\right) v^j v^k$$

$$\Delta^{ip}\frac{\partial a_i}{\partial x^p} = \Delta^{ip}\frac{\partial^2 h_{ij}}{\partial x^k \partial x^p} v^j v^k = \boxed{\frac{d}{ds}\Delta^{ip}\frac{\partial h_{ij}}{\partial x^p} v^j}$$

$$\cancel{\Delta^{ip}\frac{\partial}{\partial x^p}\frac{d}{ds} v_i}$$

$$\cancel{\frac{d}{ds}\frac{\partial}{\partial x^p}}$$

$$\cancel{\frac{d}{ds}\Delta^{ip}\frac{\partial v_i}{\partial x^p} = \frac{d}{ds}\Delta^{ip}\frac{\partial h_{ij}}{\partial x^p} v^j}$$

$$\cancel{\frac{\partial v_i}{\partial x^p} - \frac{\partial h_{ij}}{\partial x^p} v^j = C_{ip} \quad \text{constante}}$$

$$h_{ij} v^j$$

$$\boxed{\Delta^{ip}\frac{\partial^2 h_{jk}}{\partial x^i \partial x^p} v^j v^k = 0}$$

$$\Delta^{ip}\frac{\partial^2 h_{jk}}{\partial x^i \partial x^p} v^j v^k = 0$$

$$\delta\int_A^B \frac{\partial h_{jk}}{\partial x^i} v^j dx^k = 0$$

$$ds = \sqrt{\Delta_{mn}\, dx^m dx^n}$$

$$\delta(ds) = \frac{\Delta_{mn}\, dx^m d(\delta x^n)}{ds}$$

$$\delta\int_A^B \frac{\partial h_{jk}}{\partial x^i} v^j dx^k = 0$$

$$\delta(ds) = v_n\, d(\delta x^n)$$

$$v^j = \frac{dx^j}{ds}$$

$$\delta v^j = \frac{d(\delta x^j)}{ds} - \frac{dx^j v_n\, d(\delta x^n)}{ds^2}$$

$$\delta\int_A^B \frac{\partial h_{jk}}{\partial x^i} v^j dx^k = \int_A^B \frac{\partial^2 h_{jk}}{\partial x^i \partial x^p} v^j dx^k \delta x^p$$

$$+ \int_A^B \frac{\partial h_{jk}}{\partial x^i} v^j d(\delta x^k) - \int_A^B \frac{\partial h_{jk}}{\partial x^i} v^j v^k v_n\, d(\delta x^n)$$

$$= \int_A^B \left(\frac{\partial^2 h_{jk}}{\partial x^i \partial x^p} v^j dx^k \delta x^p - dx^k \frac{d}{ds}\left\{\frac{\partial h_{jk}}{\partial x^i} v^j\right\} + \frac{d}{ds}\left\{\frac{\partial h_{jk}}{\partial x^i} v^j v^k v_n\right\} \delta x^n \right.$$

$$\boxed{\Delta^{pq} \frac{\partial^2 h_{ij}}{\partial x^p \partial x^q} v^i v^j}$$

$$\delta \int_A^B \frac{\partial h_{ij}}{\partial x^p} \boxed{\cancel{v^i v^j v^p}}$$

$\nu_1 + \nu_2 = -2\mu$

$$(1 - 2\mu z + z^2) = (1 - \nu_1 z)(1 - \nu_2 z)$$

$$(1 - 2\mu z + z^2) = (1 - \nu z)(1 - \nu^{-1} z)$$

$$\{1 - 2\mu z + z^2\}^{-\frac{1}{2}} = (1 - \nu z)^{-\frac{1}{2}} (1 - \nu^{-1} z)^{-\frac{1}{2}}$$

$$\nu = \mu \pm \sqrt{\mu^2 - 1}$$

$$(1 - 2\mu z + z^2) = 0$$

$$z = \mu \pm \sqrt{\mu^2 - 1}$$

$$(1 - 2\mu z + z^2) = (z - \nu_1)(z - \nu_2)$$

$$\left.\begin{aligned} \nu_1 &= \mu + \sqrt{\mu^2 - 1} \\ \nu_2 &= \mu - \sqrt{\mu^2 - 1} \end{aligned}\right\} \mu_1 \mu_2 = +1$$

$$\nu_1 \nu_2 = \mu^2 - \mu^2 + 1 = 1$$

$$(1-\nu z)^{-\frac{1}{2}} = 1 + \frac{1}{2}\nu z + \frac{1\cdot 3}{2\cdot 4}\nu^2 z^2 + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\nu^3 z^3 + \frac{1\cdot 3\cdot 5\cdot 7}{2\cdot 4\cdot 6\cdot 8}\nu^4 z^4 +$$

$$(1-\nu^{-1} z)^{-\frac{1}{2}} = 1 + \frac{1}{2}\nu^{-1} z + \frac{1\cdot 3}{2\cdot 4}\nu^{-2} z^2 + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\nu^{-3} z^3 + \frac{1\cdot 3\cdot 5\cdot 7}{2\cdot 4\cdot 6\cdot 8}\nu^{-4} z^4 .$$

$$= 1 + \nu^{-1} z + \left[\frac{1\cdot 3}{2\cdot 4}\nu^2 + \frac{1\cdot 1}{2\cdot 2} + \frac{1\cdot 3}{2\cdot 4}\nu^{-2}\right] z^2 + \left[\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\nu^3 + \frac{1}{2}\cdot\frac{1\cdot 3\cdot}{2\cdot 4}\nu + \frac{1}{2}\frac{1\cdot 3}{2\cdot 4}\right.$$

$$\nu z^4 + \frac{1\cdot3\cdot5\cdot7\cdot9}{2\cdot4\cdot6\cdot8\cdot10}\nu^5 z^5 + \ldots$$

$$\nu^{-4} z^4 + \frac{1\cdot3\cdot5\cdot7\cdot9}{2\cdot4\cdot6\cdot8\cdot10}\nu^{-5} z^5 + \ldots$$

$$\nu + \frac{1}{2}\frac{1\cdot3}{2\cdot4}\nu^{-2} + \frac{1\cdot3\cdot5}{2\cdot4\cdot6}\nu^{-3}\Big] z^3$$